Ans 1: As per EHS theory, Reissner (1936)

$$b = \frac{m}{\rho r_0^3}$$

& $$a_{on} = \omega_n r_0 \sqrt{\frac{\rho}{G}}$$

$$ba_{on}^2 = \frac{m}{\cancel{\rho}\, r_0^{\cancel{3}}} \cdot \omega_n^2 \cancel{r_0^{2}} \cdot \frac{\cancel{\rho}}{G}$$

$$= \frac{m\omega_n^2}{G r_0}$$

A/C to Timoshenko & Goodier (1951).

$$K = \frac{4 G r_0}{1-\mu} \quad \& \quad \omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore\ ba_{on}^2 = \frac{\cancel{m}\cdot\left(\frac{K}{\cancel{m}}\right)}{Gr_0}$$

$$= \frac{K}{Gr_0}$$

$$= \frac{4\,\cancel{Gr_0}}{(1-\mu)\cdot\cancel{Gr_0}}$$

$$\boxed{ba_{on}^2 = \frac{4}{1-\mu}}$$

$\mu = 0.333$

$$\therefore b a_{on}^2 = \frac{4}{1 - 0.333}$$

$$= 6.0$$

(Proved).

Ans 2:— We know,

$$k_z = C_u . A$$

& $$k_x = C_z . A$$

$$\therefore \frac{k_z}{k_x} = \frac{C_u}{C_z}$$

Now, $$k_z = \frac{4 G r_0}{1-\mu}$$

$$K_x = \frac{32(1-\mu)G r_0}{7-8\mu}$$

a/c to Bycroft (1956)

$$\frac{C_u}{C_z} = \frac{\cancel{4}\,\cancel{G r_0}}{(1-\mu)} \cdot \frac{(7-8\mu)}{\underset{8}{\cancel{32}}(1-\mu)\cancel{G r_0}}$$

$$\therefore \frac{C_u}{C_z} = \frac{(7-8\mu)}{8(1-\mu)^2}$$

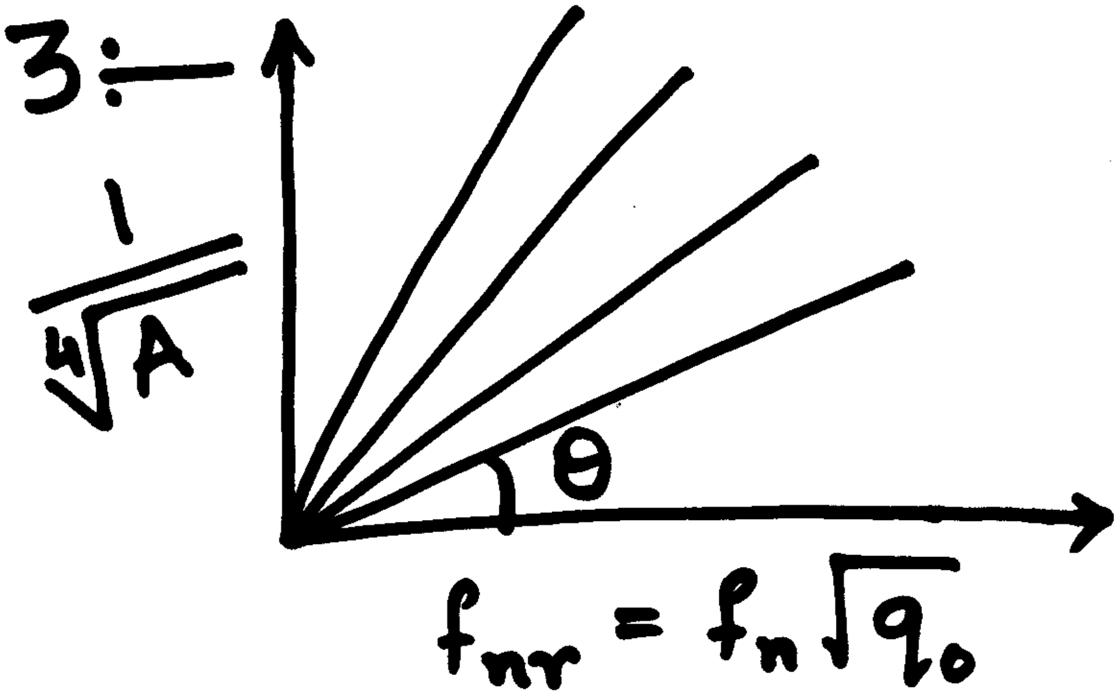
$$\mu = 0{\cdot}45$$

$$\therefore \quad \frac{C_u}{C_z} = \frac{7 - 8(0{\cdot}45)}{8(1 - 0{\cdot}45)^2}$$

$$\therefore \quad \boxed{C_u = 1{\cdot}4\, C_z}$$

(Proved).

Ans 3:-



We need to prove that

$$\tan\theta \propto \sqrt{\frac{\pi^{2.5}(1-\mu)}{Gg}}$$

Now, $b = \dfrac{m}{\rho r_0^3}$

$$= \frac{mg}{(\rho g) r_0^3}$$

$$= \frac{W}{\gamma r_0^3}$$

$$= \frac{W}{\frac{\gamma r_0}{\pi} \cdot \pi r_0^2} = \frac{W}{A} \cdot \frac{\pi}{\gamma r_0}$$

$$= \frac{q_0 \pi}{\gamma r_0}$$

$$a_{on} = \omega_n r_o \sqrt{\frac{\rho}{G}}$$

$$= 2\pi f_n r_o \sqrt{\frac{\rho}{G}}$$

$$\therefore \quad a_{on}^2 = 4\pi^2 f_n^2 \cdot r_o^2 \cdot \frac{\rho}{G}$$

$$b a_{on}^2 = \frac{4\pi^3 r_o f_n^2}{g G} \cdot q_o$$

$$= \frac{4\pi^{2\cdot5}\sqrt{A}}{g G} \cdot q_o f_n^2$$

$$\therefore \; f_n^2 q_0 = \frac{Gg}{4\pi^{2.5}\sqrt{A}} \cdot b a_{on}^2$$

$$\therefore \; f_n \sqrt{q_0} = \sqrt{\frac{G g b a_{on}^2}{4\pi^{2.5}}} \cdot \frac{1}{\sqrt[4]{A}}$$

Vertical mode of vibration,

$$b a_{on}^2 = \frac{4}{1-\mu}$$

$$f_n\sqrt{q_0} = \sqrt{\frac{Gg\cdot 4}{4\pi^{2\cdot5}(1-\mu)}}\cdot\frac{1}{\sqrt[4]{A}}$$

$$\text{or, } \frac{1}{\sqrt[4]{A}} = \sqrt{\frac{\pi^{2\cdot5}(1-\mu)}{Gg}}\cdot f_{nr}$$

$$\therefore \tan\theta \propto \sqrt{\frac{\pi^{2\cdot5}(1-\mu)}{Gg}}.$$

(Proved)

Ans 4:— $N_{60} = 14$

$a_{max} = 0.24g$

Assume, $\gamma_{sat} = \gamma = 17\ kN/m^3$

above GWT.

$Z = 10\ m$

$\sigma_v = 17 \times 10 = 170\ kN/m^2$

$\sigma_v' = (17 \times 5) + \{(17 - 10) \times 5\}$

$= 120\ kN/m^2 > 100\ kPa.$

As per Youd et al. (2001),

$$r_d = 1.174 - 0.0267z$$

$$= 1.174 - 0.0267(10)$$

$$= 0.907$$

$$CSR = 0.65 \cdot \left(\frac{a_{max}}{g}\right)\left(\frac{\sigma_v}{\sigma_v'}\right) \cdot r_d$$

$$= 0.65\left(\frac{0.24\cancel{g}}{\cancel{g}}\right)\left(\frac{170}{120}\right)(0.907)$$

$$= 0.20045$$

SPT N_{60},

$$C_N = \left(\frac{P_a}{\sigma_v'}\right)^{0.5}$$

$$= \left(\frac{100}{120}\right)^{0.5} = 0.913$$

Assume, C_B, C_R & $C_S = 1.0$

$$(N_1)_{60} = 14 \times 0.913 = 12.8$$

Now, $FC = 6\% \begin{matrix} > 5\% \\ < 35\% \end{matrix}$

$$(N_1)_{60,cs} = \alpha + \beta (N_1)_{60}$$

$$\alpha = \exp\left[1{\cdot}76 - \left(\frac{190}{6^2}\right)\right]$$

$$= 0{\cdot}0297$$

$$\beta = 0{\cdot}99 + \left(\frac{6^{1{\cdot}5}}{1000}\right)$$

$$= 1{\cdot}0047$$

$$(N_1)_{60,cs} = 0.0297 + 1.0047(12.8)$$

$$= 12.9 < 30$$

$$(CRR)_{7.5} = \frac{1}{34-(N_1)_{60}} + \frac{(N_1)_{60}}{135} + \frac{50}{[10(N_1)_{60}+45]^2} - \frac{1}{200}$$

$$(CRR)_{7.5} = 0.1396$$

$$K_M = \frac{10^{2.24}}{M_w^{2.56}}$$

$$= \frac{10^{2.24}}{(6.5)^{2.56}}$$

$$= 1.442$$

$$K_\sigma = \left(\frac{\sigma_v'}{P_a}\right)^{(f-1)}$$

$$K_\sigma = \left(\frac{120}{100}\right)^{(0.7-1)}$$

$$= 0.9468$$

Assume. $K_\alpha = 1.0$

$$FS_L = \frac{(CRR)_{7.5}}{CSR} \cdot K_M \cdot K_\sigma \cdot K_\alpha$$

$$= \frac{0.1396}{0.20045}(1.442)(0.9468)(1.0)$$

$$= 0.95 < 1.0 \quad \underline{\text{Ans.}}$$

$$< 1.3$$